SIMULATION OF RADAR SCATTERING FROM ROUGH SURFACES

Final Report

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SUMMARY

The objective of this project was to establish computer simulation as a tool for studying scattering from randomly rough surfaces. The proposed methods to be investigated included the standard moment method and the integral equation method (IEM). The project began by investigating the

was investigated for

minimum size ~~of~~ the surface patch necessary to obtain the scattering cross section per unit area, which should be the same as obtained from an infinite surface patch when the method used was the standard moment method. ~~It was found that~~ five points per wavelength, or per surface correlation length (whichever was the smaller), were needed and the minimum surface patch should be at least seven electric wavelengths, or surface correlation lengths (whichever was the larger). The minimum number of surface patches to be averaged to obtain a satisfactory estimate of the scattering coefficient was 25. This means that the smallest number of unknowns to be evaluated was 35^2 for a scaler problem and 2 times 35^2 for a vector problem. To determine this many unknowns for a scalar problem one needs to invert a matrix size of 35^2 by 35^2 in the moment method. In general, the size of the problem would be larger and hence the integral equation method was investigated. ~~It was found that~~ the IEM was a better method than the moment method in that it required much less memory. It required a much greater computational time, ~~however~~. For this reason another method called the frequency domain method (FDM) was considered towards the end of the project. The FDM was the preferred method as compared with the moment method and the IEM in two-dimensional surface scattering problems because it was more efficient in both memory and computational time. Due to time limitation,

extensions of the FDM to three dimensional scattering problems are yet to be done.

1. INTRODUCTION

This project was started in August 1987 but the funding was discontinued in February 1988. It was restarted in January 1989 for a period of one year. The proposed studies were completed and, in addition, a new approach, the frequency domain method, was also investigated. The chronological progress of the project is given in the next paragraph and accomplished studies are given in the next section with reference to the reports delivered during the course of the project. The work done over the last quarter is given in Section 3. A conclusion is given in Section 4.

The aim of this project was to establish computer simulation of radar scattering as a tool for investigating surface scattering phenomena by defining the scope of the problem and the technique to be used. At the time the proposal was being prepared the standard method for simulating surface scattering was the method of moment. The use of the moment method requires the inversion of a matrix of size proportional to the square of the number of unknowns. In surface scattering the number of unknowns is equal to the number of surface points needed to define the surface patch from which we want to compute scattering. Thus, for a surface with 100 by 100 points we have a matrix of size 10^4 x 10^4, which is clearly too large for today's supercomputer. Hence, an immediate interest is to determine how small a surface patch we need to simulate the scattering cross section per unit

area from an infinite patch. The immediate goal was to find the answers to the following:

(a) What was the minimum size of the surface patch in terms of the number of surface correlation lengths or the electrical wavelengths--whichever was larger?

(b) What was the minimum number of points per electrical wavelength or surface correlation length--whichever was smaller?

In the event that the minimum requirement for the number of unknowns came out to be larger than what was suitable for the moment method, it was proposed that the integral equation method (IEM) should be investigated. It was demonstrated that IEM was a viable method which could be handled by to-day's computer. However, it took a lot of computer time. Finally, a frequency domain method was investigated and demonstrated to be efficient in both memory and computational time for a two-dimensional scattering problem. The theoretical basis for its extension was developed during the last period of this project and is given in Section 3.

2. STUDIES ACCOMPLISHED

Surface scattering simulation begins with a computer-generated, statistically known surface to which we apply our algorithm to perform scattered field and power computations. In what follows we list the major tasks accomplished and the report in which details were given.

(a) Generation of a statistically known random surface

This task was reported in the quarterly report for the period, August to October 1987. It was shown that one can specify the surface height statistics and its correlation function and that the algorithm would then generate surface patches with the specified statistical properties.

(b) Specification of minimum simulation parameters

The minimum number of surface points per electrical wavelength or per correlation length (whichever is the smaller) was reported to be five in the quarterly report for the period August to October 1987. The minimum effective width of the antenna pattern was found to be at least three times the surface correlation length. One must integrate the antenna pattern until it dropped to 10^{-3} of its value along the boresight. Additional studies on simulation parameters were reported in the quarterly report for the period November to January 1988. There it was indicated that to obtain an acceptable estimate of the scattering coefficient one needed to average at least 25 samples. It was also found that the minimum width of the illuminated area should be either seven surface correlation lengths or seven electrical wavelengths--whichever was the larger.

(c) Integral equation method

The issue of rough surface scattering depends on obtaining an accurate representation of the surface current. Once the surface current is known only a two-dimensional integration has to be performed to arrive at the scattered field. It is well known that the surface current is defined by an integral equation. Hence, an attempt was made to solve this equation and arrive at an estimate for the surface current (see quarterly report for the period February to May 1989). We shall refer to this approach as the

integral equation method. To verify the applicability of this estimate it is best to compare scattering coefficients based on it with those based on the moment method. Since in practice we cannot use the moment method in a three-dimensional surface scattering problem due to its excessive matrix size, we have performed scattering calculations for two-dimensional scattering problems. This was done and reported in the quarterly report for the period February May 1989. The comparisons between the two approaches indicate that except for surfaces with rms slopes larger than 0.4 the scattering coefficient obtained using the integral equation method is accurate at all frequencies and for all incidence angles from nadir to 60 degrees from the vertical. In view of this success the same concept to obtain a current estimate is used in three-dimensional surface scattering problems. It was shown in our quarterly report for the period February to May 1989 that simulated results could be obtained and compared with measurements as well as theoretical predictions. The only drawback with the integral equation method is that it takes considerable amount of computer time to do the three-dimensional surface scattering problem because one has to evaluate a four-fold integral.

(d) Frequency domain method

In an effort to find a more efficient approach we have developed a frequency domain method (described in the quarterly report for the period from May to August 1989) where we use the standard spectral representation for the scattered field and the boundary condition as the integral equation to be solved. When applied to a two-dimensional surface scattering problem considerable saving was obtained: what takes an hour on a VAX 8700

computer using moment method was reduced to 3 minutes. Efforts are currently underway to extend this method to three-dimensional problems.

3. FREQUENCY DOMAIN METHOD IN 3-D SCATTERING

The theoretical basis for the frequency domain technique in a three-dimensional scattering problem is given below. The concept is the same as in a two-dimensional problem. Thus, the form of the scattered field is taken to be

$$E^s = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \, \alpha(k_x, k_y) \, e^{-jk_x x + jk_y y - jk_z \cdot z}$$

and it should satisfy the tangential boundary condition, $\hat{n} x \, \overline{E} = 0$. For a y-polarized field incident in the xz-plane, the x-component of the tangential field is

$$E_y^i + E_y^s + Z_y \, E_z^s = 0$$

Since surface depolarization is usually more than an order of magnitude smaller than the like and surface slope is usually less than unity, for like polarization we can drop the third term in the above equation and write

$$e^{-jk_{xo} x + jk_{zo} z(x,\, y)} = -\int \int \alpha\left(k_x, k_y\right) e^{-jk_x x + jk_y y - jk_z \cdot z(x,\, y)} \, dk_x dk_y \tag{1}$$

In (1) the expression on the left hand side is the incident field and α is the unknown to be determined. To do so we multiply both sides of (1)by $e^{-jk_{xo}x-jk_{yo}y+jk_{zo}z(x,\,y)}$ and the pattern function, G (x,y) and integrate over the illuminated area yielding

$$\int\int e^{-j2k_{xo}x-jk_{yo}y+j2k_{zo}z(x,\,y)}G(x,\,y)\,dxdy$$

$$=-\int\int\int\int \alpha(k_x,\,k_y)e^{j(k_x x-k_{xo})x+j(k_y-k_{yo})y-j(k_z-k_{zo})z(x,\,y)}G(x,\,y)\,dxdy\,dk_x dk_y \tag{2}$$

Let

$$I(k_x,\,k_y)=\int\int e^{j(k_x-k_{xo})x+j(k_y-k_{yo})y-j(k_z-k_{zo})z(x,\,y)}G(x,\,y)\,dxdy$$

Note that $I(k_x,\,k_y)$ is a function which has a sharp peak around k_{ox} and k_{oy}. Hence, we can rewrite the right hand side of (2) as

$$\int\int \alpha(k_x,\,k_y)\,I(k_x,\,k_y)\,dk_x\,dk_y \cong \alpha(k_{xo},\,k_{yo})\int\int I(k_x,\,k_y)\,dk_x dk_y \tag{3}$$

and

$$\alpha(k_{xo},\,k_{yo})=-\frac{1}{I_o}\int\int e^{-jzk_{xo}x-jk_{yo}y+jzk_{zo}z(x,y)}G(x,y)\,dxdy \tag{4}$$

where $I_o \triangleq \int\int I(k_x,\,k_y)\,dk_x dk_y$

Next, we need to find the relationship between the scattering coefficient and α.

To do so, let us consider the ensemble average of the following product

$$\left\langle \alpha\left(k_x, k_y\right) \alpha\left(k'_x, k'_y\right)^* \right\rangle = \left(\frac{1}{I_o}\right)^2 \int\int\int\int \left\langle e^{-jzk_x x - jzk_y y + jzk_z z(x,y)} G(x,y)\, dxdy \right.$$

$$\left. e^{jzk'_x x' + jzk'_y y' - jzk'_z z(x',y')} G(x',y')\, dx'dy' \right\rangle$$

$$= \left(\frac{1}{I_o}\right)^2 \int\int\int\int e^{-jzk_x(x-x') + jz(k_x - k'_x)\,x' - jzk_y(y-y') + jz\,(k_y - k'_y)}$$

$$e^{-4\sigma^2\left(k_z^2 + k'^2_z - 2k_z k'_z\,\rho\right)/2} G\,(x,\ y)\, G\,(x',y')\, dxdy'\ dydy'$$

$$= \left(\frac{1}{I_o}\right)^2 \int\int\int\int e^{-jzk_x\,\xi + jz\,\Delta\,k_x\,x' - jzk_y\,\eta + jz\,\Delta k_y\,y'}$$

$$e^{-2\sigma^2\left(k_z^2 + k_z'^2 - 2k_z k'_z\,\rho\,(\xi,\,\eta)\right)} G\,(\xi + x',\ \eta + y')\, G\,(x',y')\, dx'dy'\ d\xi d\eta' \tag{5}$$

where ρ is the surface correlation function and σ is the surface rms height.

If $k_x = k'_x$, $k_y = k'_y$, $k_z = k'_z$, we obtain another expression,

$$\left\langle \alpha\left(k_x, k_y\right) \alpha^*\left(k_x, k_y\right) \right\rangle = \left\langle \left| \alpha\ \left(k_x, k_y\right) \right|^2 \right\rangle$$

$$= \left(\frac{1}{I_o}\right)^2 \int\int\int\int e^{-jzk_x\,(x - x') - jzk_y\,(y - y') - (2k_z\,\sigma)^2\,(1 - \rho)} G\,(x,\ y)\, G\,(x',\ y')\, dxdxdydy'$$

$$= \left(\frac{1}{I_o}\right)^2 \int\int\int\int e^{-jzk_x\xi - jzk_y\eta - (zk_z\sigma)^2(1-\rho(\zeta,\eta))} G(\zeta + x', \eta + y')\, G(x', y')\, d\zeta d\eta dx' dy'$$

(6)

Let us assume that the pattern function is Gaussian,

$$G(x, y) = e^{-(x^2+y^2)/g^2}$$

Then, the following integrals given below by (7) and (8) can be evaluated

$$\int\int G(x+\xi, y+\eta)\, G(x, y)\, dxdy = \int\int e^{-\frac{(x+\zeta)^2}{g^2} - \frac{x^2}{g^2} - \frac{(y+\eta)^2}{g^2} - \frac{y^2}{g^2}} dxdy$$

$$= e^{-\frac{\zeta^2}{2g^2}} \int e^{-2\left(x+\frac{\zeta}{2}\right)^2/g^2} dx\, e^{-\frac{\eta^2}{2g^2}} \int e^{-2\left(y+\frac{\eta}{2}\right)^2/g^2} dy$$

$$\cong e^{-\frac{\zeta^2+\eta^2}{2g^2}} \left(g\sqrt{\frac{\pi}{2}}\right)^2 = \frac{g^2\pi}{2}\, e^{-\frac{\xi^2+\eta^2}{2g^2}}$$

(7)

$$\int\int e^{jz\Delta k_x x + jz\Delta k_y y} G(\xi+x, \eta+y)\, G(x, y)\, dxdy$$

$$\cong \frac{g^2\pi}{2}\, e^{\left[-\frac{\xi^2}{2g^2} - j\Delta k_x\xi - \frac{g^2\Delta k_x^2}{2}\right]} e^{\left[-\frac{\eta^2}{2g^2} - j\Delta k_y\eta - \frac{g^2\Delta k_y^2}{2}\right]}$$

(8)

Using (7) and (8) we can rewrite (5) and (6) as (9) and (10).

$$\left\langle \alpha(k_x)\, \alpha(k'_x)^* \right\rangle = \left(\frac{1}{I_o}\right)^2 g^2 \frac{\pi}{2} \int\int e^{-jzk_x\xi - j2k_y\eta - 2\sigma^2\left(k_z^2 + k_z'^2 - 2k_z k'_z \rho(\xi,\eta)\right)}$$

$$e^{-\frac{\zeta^2}{2g^2} - j\Delta k_x \xi - \frac{g^2 \Delta k_x^2}{2} - \frac{\eta^2}{2g^2} - j\Delta k_y \eta - \frac{g^2 \Delta k_y^2}{2}} d\xi d\eta \tag{9}$$

$$\left\langle |\alpha(k_x)|^2 \right\rangle = \left(\frac{1}{I_o}\right)^2 g^2 \frac{\pi}{2} \int \int e^{-j2k_x\xi - j2k_y\eta - (2k_z\sigma)^2(1-\rho(\xi,\eta))} e^{-\frac{\xi^2+\eta^2}{g^2}} d\xi \, d\eta \tag{10}$$

Now we are ready to calculate the ensemble average of the scattered field with the aim of expressing it in terms of (10) as follows

$$\left\langle |E^s|^2 \right\rangle = \int \int \int \int \left\langle \alpha(k_x, k_y)\, \alpha(k'_x, k'_y)^* \right\rangle e^{j(k_x - k'_x)x + j(k_y - k'_y) - j(k_z - k'_z)z} dk_x dk'_x dk_y dk'_y \tag{11}$$

Substitute (9) into (11) and integrate k_x' and k_y' out ,we have

$$\left\langle |E^s|^2 \right\rangle = \left(\frac{1}{I_o}\right)^2 g^2 \frac{\pi}{2} \cdot \left(\frac{2\sigma}{g^2}\right) \int \int \int \int e^{j2k_x\zeta - j2k_y\eta - (2k_z\sigma)^2(1-\rho(\zeta,\eta))}$$
$$\exp\left[-(\xi^2 + \eta^2)/2g^2\right] d\zeta \, d\eta \, dk_x \, dk_y \tag{12}$$

Substituting (10) into (12) we have

$$\left\langle |E^s|^2 \right\rangle = \frac{2\pi}{g^2} \int \left\langle |\alpha(k_x, k_y)|^2 \right\rangle dk_x \, dk_y \tag{13}$$

From Ulaby et al (1982) Chapter 12, the scattering coefficient is related to the integrand of (13) as follows

$$\sigma^o = 4\pi k_s^2 \cos^2\theta_s \frac{2\pi}{g^2} \left\langle |\alpha(k_x, k_y)|^2 \right\rangle \tag{14}$$

Since α is given by (4) we can evaluate the scattering coefficient.

4. CONCLUSIONS

We have accomplished all the tasks proposed and gone beyond them by investigating new approaches as shown in the latter part of Section 2 and Section 3.